Technical Appendix (For Online Publication) "Bond Finance, Bank Credit, and Aggregate Fluctuations in an Open Economy"

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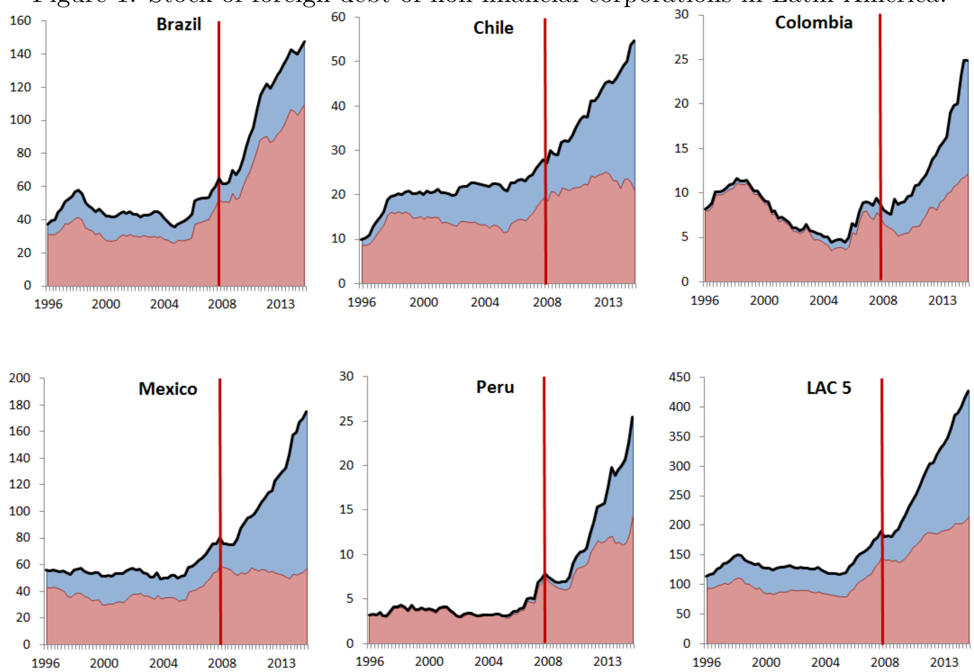
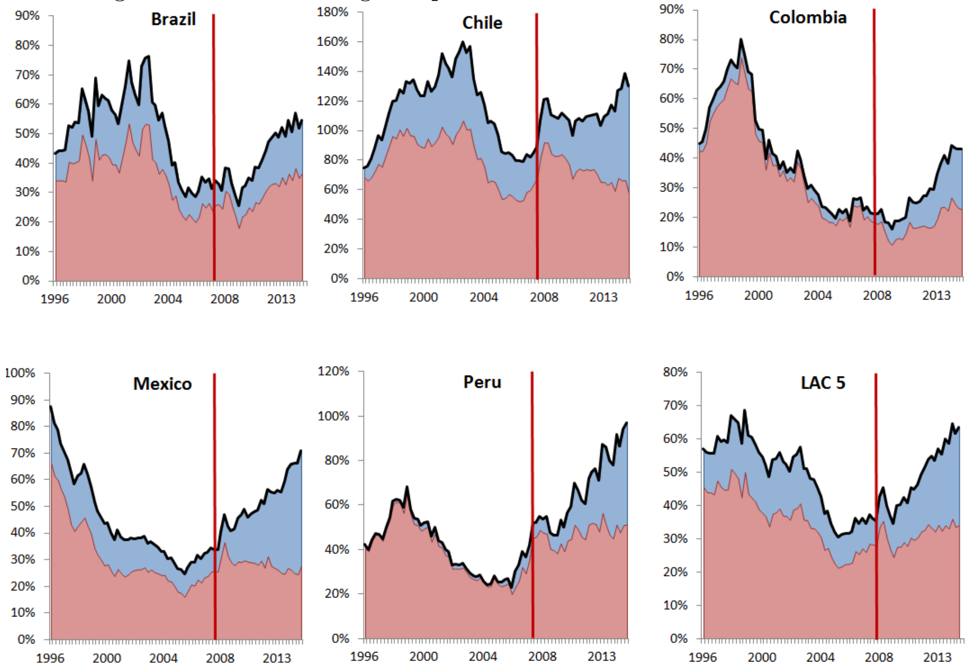
1 Empirical facts

Figure 1: Stock of foreign debt of non-financial corporations in Latin America.



Notes: Units are billion USD. Red (bottom) areas indicate the stock of outstanding bank loans and blue (top) areas indicate outstanding corporate bonds, both measured on a residence basis. Data for non-finanical corporations only. Vertical lines indicate the collapse of Lehman Brothers. *Sources:* Powell (2014) and BIS.

Figure 2: Stock of foreign corporate debt to GDP in Latin America.



Notes: Units are fraction of GDP (in percent). Red (bottom) areas indicate the stock of outstanding bank loans and blue (top) areas indicate outstanding corporate bonds, both measured on a residence basis. Vertical lines indicate the collapse of Lehman Brothers. *Sources:* Powell (2014) and BIS.

2 Benchmark model

First, note that you can define a small model that easily closes. The model consists of the following 29 variables, all known at time t: D_{t+1}, $\bar{D}_{t+1}$, Ψ_{t+1}, C_t, H_t, W_t, λ_t^h, Y_t, K_{t+1}, r_t^K, Q_t, I_t, i_t, $\bar{A}_t$, $\underline{A}_t$, $\bar{a}_t$, $\underline{a}_t$, λ_t^1, λ_t^2, X_t, Π_t^f, β_t, μ_{t+1}, R_t^m, I_t^m, K_{t+1}^m, K_{t+1}^f, CB_t and BL_t.

The complete list of 29 equations is:

$$\Psi_t = \bar{\Psi} + \tilde{\Psi}\left(\mathrm{e}^{\bar{D}_t - \bar{D}} - 1\right) \tag{1}$$

$$\bar{D}_{t+1} = D_{t+1} \tag{2}$$

$$C_t + Q_t X_t + \Psi_t R_t^* D_t = W_t H_t + r_t^K K_t + \left(1 - \phi^f\right) \Pi_t^f + D_{t+1} \tag{3}$$

$$\lambda_t^h = \beta^h \Psi_{t+1} R_{t+1}^* E_t \lambda_{t+1}^h \tag{4}$$

$$\lambda_t^h = \left(C_t - \kappa \frac{H_t^\tau}{\tau}\right)^{-\sigma} \tag{5}$$

$$\kappa H_t^{\tau-1} = W_t \tag{6}$$

$$Y_t = Z K_t^\alpha H_t^{1-\alpha} \tag{7}$$

$$(1-\alpha)\frac{Y_t}{H_t} = W_t \tag{8}$$

$$\alpha \frac{Y_t}{K_t} = r_t^K \tag{9}$$

$$K_{t+1}^m = \phi^m \frac{p_H c I_t}{\Delta}\left[G\left(\bar{A}_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \tag{10}$$

$$K_{t+1} = (1-\delta)K_t + X_t - \frac{\varphi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{11}$$

$$X_t = p_H R I_t \left[1 - G\left(\underline{A}_t;\mu_t\right)\right] \tag{12}$$

$$Q_t\left[1 + \varphi\left(\frac{K_{t+1}}{K_t} - 1\right)\right] = E_t \beta^h \frac{\lambda_{t+1}^h}{\lambda_t^h}\left\{r_{t+1}^K + Q_{t+1}\left[1 - \delta + \varphi\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\frac{K_{t+2}}{K_{t+1}} - \frac{\varphi}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)^2\right]\right\} \tag{13}$$

$$K_{t+1}^f = \phi^f \Pi_t^f \tag{14}$$

$$\Pi_t^f = \left(p_H Q_t R - 1\right) I_t \left[1 - G\left(\underline{A}_t;\mu_t\right)\right] + K_t^f - \frac{p_H c I_t}{\Delta\beta_t}\left(\beta_t - 1\right)\left[G\left(\bar{A}_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \tag{15}$$

$$\begin{aligned} &\left(p_H Q_t R - 1\right)\left[1 - G\left(\underline{A}_t;\mu_t\right)\right] - \frac{p_H c}{\Delta\beta_t}\left(\beta_t - 1\right)\left[G\left(\bar{A}_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \\ &\quad - \lambda_t^1\left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] - \lambda_t^2\left[1 - \frac{p_H c}{\Delta\beta_t} - p_H\left(Q_t R - \frac{b+c}{\Delta}\right)\right] = 0 \end{aligned} \tag{16}$$

$$\lambda_t^1 = g\left(\bar{A}_t;\mu_t\right) I_t \frac{p_H c}{\Delta\beta_t}\left(\beta_t - 1\right) \tag{17}$$

$$\lambda_t^2 = g\left(\underline{A}_t;\mu_t\right) I_t \left[\left(p_H Q_t R - 1\right) - \frac{p_H c}{\Delta\beta_t}\left(\beta_t - 1\right)\right] \tag{18}$$

$$i_t = \frac{I_t}{K_t^f} \tag{19}$$

$$\bar{A}_t = I_t\left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] \tag{20}$$

$$\underline{A}_t = I_t\left[1 - \frac{p_H c}{\Delta\beta_t} - p_H\left(Q_t R - \frac{b+c}{\Delta}\right)\right] \tag{21}$$

$$\bar{a}_t = i_t\left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] \tag{22}$$

$$\underline{a}_t = i_t\left[1 - \frac{p_H c}{\Delta\beta_t} - p_H\left(Q_t R - \frac{b+c}{\Delta}\right)\right] \tag{23}$$

$$K_t^m = I_t^m\left[G\left(\bar{A}_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \tag{24}$$

$$R_t^m = \frac{c I_t}{\Delta} \tag{25}$$

$$I_t^m = \frac{p_H R_t^m}{\beta_t} \tag{26}$$

$$\mu_{t+1} = \ln K_{t+1}^f - \frac{\sigma_G^2}{2} \tag{27}$$

$$CB_t = \int_{\bar{A}_t}^{I_t}\left(I_t - A_t^i\right) \mathrm{d}G\left(A_t^i;\mu_t\right) \tag{28}$$

$$BL_t = \int_{\underline{A}_t}^{\bar{A}_t} I_t - A_t^i \,\mathrm{d}G\left(A_t^i;\mu_t\right) \tag{29}$$

On top of that we have one exogenous variable R_{t+1}^* that is subject to the following autoregressive stochastic process:

$$\ln R_{t+1}^* = \rho_{R^*}\ln R_t^* + \left(1-\rho_{R^*}\right)\ln R^* + \epsilon_{R^*,t},\ \left|\rho_{R^*}\right| < 1,\ \epsilon_{R^*,t} \overset{i.i.d}{\sim} N\left(0,\sigma_{R^*}^2\right) \tag{30}$$

We also have the following list of 17 parameters and constants in the model: ϕ^f, ϕ^m, $\bar{\Psi}$, $\tilde{\Psi}$, β^h, κ, τ, σ, α, δ, φ, σ_G, p_H, p_L, c, b, B. This list excludes parameters in the AR equation.

3 Benchmark model - linearization

The log-linearization is computed by taking first-order Taylor expansion around the NSSS. Most deviations are defined as percentage deviations from their own steady state value, i.e., $\hat{x}_t = \frac{x_t - x}{x}$. Some variables ($D_t$ in our case) can in principle take negative values, so they are computed as deviations from steady state output Y. μ_t can also take negative values so it's deviation is defined as percentage deviation from value 1, i.e., $\hat{\mu}_t = \frac{\mu_t - \mu}{1}$. Before moving to the equations, let us also define a few useful NSSS objects. It is assumed that $G(A)$ is the lognormal CDF with parameters μ and σ_G and $g(A)$ is the corresponding PDF.

$$g_\mu\left(\bar{A}\right) = \frac{\partial}{\partial \mu} g\left(\bar{A}\right) = g\left(\bar{A}\right)\left(\frac{\ln \bar{A} - \mu}{\sigma_G^2}\right) \tag{31}$$

$$g_{\bar{A}}\left(\bar{A}\right) = \frac{\partial}{\partial \bar{A}} g\left(\bar{A}\right) = -g\left(\bar{A}\right)\left(\frac{\ln \bar{A} - \mu}{\sigma_G^2} + 1\right)\frac{1}{\bar{A}} \tag{32}$$

$$G_\mu\left(\bar{A}\right) = \frac{\partial}{\partial \mu} G\left(\bar{A}\right) = \int_0^{\bar{A}} \frac{\ln A^i - \mu}{A^i \sigma_G^3 \sqrt{2\pi}} \mathrm{e}^{-\frac{\left(\ln A^i - \mu\right)^2}{2\sigma_G^2}} \mathrm{d}A^i \tag{33}$$

The linearized model equations are the following:

$$\hat{\Psi}_{t+1} - \frac{\tilde{\Psi} Y}{\bar{\Psi}} \hat{D}_{t+1} = 0 \tag{34}$$

$$\begin{aligned} C\hat{C}_t + QX\hat{Q}_t + QX\hat{X}_t - WH\hat{W}_t - WH\hat{H}_t - r^K K \hat{r}_t^K - Y\hat{D}_{t+1} - \left(1 - \phi^f\right)\Pi^f \hat{\Pi}_t^f = \\ - \Psi R^* D \hat{\Psi}_t - \Psi R^* D \hat{R}_t^* - \Psi R^* Y \hat{D}_t + r^K K \hat{K}_t \end{aligned} \tag{35}$$

$$\hat{\lambda}_t^h - \hat{\Psi}_{t+1} - R_{t+1}^* - E_t \hat{\lambda}_{t+1}^h = 0 \tag{36}$$

$$\hat{\lambda}_t^h + \left(\lambda^h\right)^{1/\sigma} \sigma C \hat{C}_t - \left(\lambda^h\right)^{1/\sigma} \sigma \kappa H^\tau \hat{H}_t = 0 \tag{37}$$

$$\left(\tau - 1\right)\hat{H}_t - \hat{W}_t = 0 \tag{38}$$

$$\hat{Y}_t - \hat{A}_t - (1-\alpha)\,\hat{H}_t = \alpha \hat{K}_t \tag{39}$$

$$\hat{Y}_t - \hat{H}_t - \hat{W}_t = 0 \tag{40}$$

$$\hat{Y}_t - \hat{r}_t^K = \hat{K}_t \tag{41}$$

$$\left[G\left(\bar{A}\right) - G\left(\underline{A}\right)\right]\hat{K}_{t+1}^m - \left[G\left(\bar{A}\right) - G\left(\underline{A}\right)\right]\hat{I}_t - g\left(\bar{A}\right)\bar{A}\hat{\bar{A}}_t + g\left(\underline{A}\right)\underline{A}\hat{\underline{A}}_t = \left[G_\mu\left(\bar{A}\right) - G_\mu\left(\underline{A}\right)\right]\hat{\mu}_t \tag{42}$$

$$K\hat{K}_{t+1} - X\hat{X}_t = (1-\delta)\,K\hat{K}_t \tag{43}$$

$$X\hat{X}_t - X\hat{R}_t - X\hat{I}_t + p_H R I g\left(\underline{A}\right)\underline{A}\hat{\underline{A}}_t = -p_H R I G_\mu\left(\underline{A}\right)\hat{\mu}_t \tag{44}$$

$$Q\hat{Q}_t + Q\varphi\left(1+\beta^h\right)\hat{K}_{t+1} - E_t\hat{\lambda}_{t+1}^h + \hat{\lambda}_t^h - \beta^h r^K E_t\hat{r}_{t+1}^K - Q\beta^h(1-\delta)\,E_t\hat{Q}_{t+1} - Q\beta^h\varphi E_t\hat{K}_{t+2} = Q\varphi\hat{K}_t \tag{45}$$

$$\hat{K}_{t+1}^f - \hat{\Pi}_t^f = 0 \tag{46}$$

$$\begin{aligned}\Pi^f\hat{\Pi}_t^f - p_H QRI\left[1-G\left(\underline{A}\right)\right]\hat{Q}_t - p_H QRI\left[1-G\left(\underline{A}\right)\right]\hat{R}_t + \frac{p_H cI}{\Delta\beta}\left[G\left(\bar{A}\right)-G\left(\underline{A}\right)\right]\hat{\beta}_t \\ - I\left\{\left(p_H QR-1\right)\left[1-G\left(\underline{A}\right)\right] - \frac{p_H c}{\Delta\beta}\left[G\left(\bar{A}\right)-G\left(\underline{A}\right)\right](\beta-1)\right\}\hat{I}_t \\ + g\left(\underline{A}\right)\underline{A}\left[p_H QRI - I - \frac{p_H cI}{\Delta\beta}(\beta-1)\right]\hat{\underline{A}}_t + g\left(\bar{A}\right)\bar{A}\left[\frac{p_H cI}{\Delta\beta}(\beta-1)\right]\hat{\bar{A}}_t = \\ -\left\{\left(p_H QR-1\right)IG_\mu\left(\underline{A}\right) + \frac{p_H cI}{\Delta\beta}(\beta-1)\left[G_\mu\left(\bar{A}\right)-G_\mu\left(\underline{A}\right)\right]\right\}\hat{\mu}_t + K^f\hat{K}_t^f\end{aligned} \tag{47}$$

$$
\begin{aligned}
& p_H QR\left[1-G\left(\underline{A}\right)+\lambda^1+\lambda^2\right]\hat{Q}_t + p_H QR\left[1-G\left(\underline{A}\right)+\lambda^1+\lambda^2\right]\hat{R}_t \\
& \quad - g\left(\underline{A}\right)\underline{A}\left[p_H QR - 1 - \frac{p_H c}{\Delta\beta}\left(\beta-1\right)\right]\hat{\underline{A}}_t - g\left(\bar{A}\right)\bar{A}\left[\frac{p_H c}{\Delta\beta}\left(\beta-1\right)\right]\hat{\bar{A}}_t \\
& \quad - \lambda^1\frac{\bar{A}}{I}\hat{\lambda}^1_t - \lambda^2\frac{\underline{A}}{I}\hat{\lambda}^2_t - \frac{p_H c}{\Delta\beta}\left[G\left(\bar{A}\right)-G\left(\underline{A}\right)+\lambda^2\right]\hat{\beta}_t \\
& \quad = \left\{\left(p_H QR-1\right)G_\mu\left(\underline{A}\right)+\left[\frac{p_H c}{\Delta\beta}\left(\beta-1\right)\right]\left[G_\mu\left(\bar{A}\right)-G_\mu\left(\underline{A}\right)\right]\right\}\hat{\mu}_t
\end{aligned}
\tag{48}
$$

$$
g\left(\bar{A}\right)\hat{\lambda}^1_t - g_{\bar{A}}\left(\bar{A}\right)\bar{A}\hat{\bar{A}}_t - g\left(\bar{A}\right)\hat{I}_t - \frac{g\left(\bar{A}\right)}{\beta-1}\hat{\beta}_t = g_\mu\left(\bar{A}\right)\hat{\mu}_t
\tag{49}
$$

$$
g\left(\underline{A}\right)\lambda^2\hat{\lambda}^2_t - g_{\underline{A}}\left(\underline{A}\right)\underline{A}\lambda^2\hat{\underline{A}}_t - g\left(\underline{A}\right)\lambda^2\hat{I}_t - \left[g\left(\underline{A}\right)\right]^2 Ip_H RQ\hat{Q}_t - \left[g\left(\underline{A}\right)\right]^2 Ip_H RQ\hat{R}_t + \frac{\left[g\left(\underline{A}\right)\right]^2 Ip_H c}{\Delta\beta}\hat{\beta}_t = g_\mu\left(\underline{A}\right)\lambda^2\hat{\mu}_t
\tag{50}
$$

$$
\hat{i}_t - \hat{I}_t = \hat{K}^f_t
\tag{51}
$$

$$
\bar{A}\hat{\bar{A}}_t - \bar{A}\hat{I}_t + Ip_H QR\hat{Q}_t + Ip_H QR\hat{R}_t = 0
\tag{52}
$$

$$
\underline{A}\hat{\underline{A}}_t - \underline{A}\hat{I}_t + Ip_H QR\hat{Q}_t + Ip_H QR\hat{R}_t - \frac{Ip_H c}{\Delta\beta}\hat{\beta}_t = 0
\tag{53}
$$

$$
\bar{a}\hat{\bar{a}}_t - \bar{a}\hat{i}_t + ip_H QR\hat{Q}_t + ip_H QR\hat{R}_t = 0
\tag{54}
$$

$$
\underline{a}\hat{\underline{a}}_t - \underline{a}\hat{i}_t + ip_H QR\hat{Q}_t + ip_H QR\hat{R}_t - \frac{ip_H c}{\Delta\beta}\hat{\beta}_t = 0
\tag{55}
$$

$$
K^m\hat{I}^m_t + I^m\bar{A}g\left(\bar{A}\right)\hat{\bar{A}}_t - I^m\underline{A}g\left(\underline{A}\right)\hat{\underline{A}}_t = K^m\hat{K}^m_t - I^m\left[G_\mu\left(\bar{A}\right)-G_\mu\left(\underline{A}\right)\right]\hat{\mu}_t
\tag{56}
$$

$$
\hat{R}^m_t - \hat{I}_t = 0
\tag{57}
$$

$$
\hat{I}^m_t - \hat{R}^m_t + \hat{\beta}_t = 0
\tag{58}
$$

$$\hat{\mu}_{t+1} - \hat{K}^{f}_{t+1} = 0 \tag{59}$$

$$CB\hat{C}B_t - \left[G\left(I\right) - G\left(\bar{A}\right)\right] I\hat{I}_t + \left(I - \bar{A}\right) g\left(\bar{A}\right) \bar{A}\hat{\bar{A}}_t = \int_{\bar{A}}^{I} \left(I - A^i\right) g_\mu\left(A^i, \mu\right) \mathrm{d}A^i \hat{\mu}_t \tag{60}$$

$$\begin{aligned} BL\hat{B}L_t - \left(I - \bar{A}\right) g\left(\bar{A}\right) \bar{A}\hat{\bar{A}}_t + \left(I - \underline{A}\right) g\left(\underline{A}\right) \underline{A}\hat{\underline{A}}_t - \left[G\left(\bar{A}\right) - G\left(\underline{A}\right)\right] I\hat{I}_t = \\ = \left\{ I \int_{\underline{A}}^{\bar{A}} g_\mu\left(A^i, \mu\right) \mathrm{d}A^i - \int_{\underline{A}}^{\bar{A}} A^i g_\mu\left(A^i, \mu\right) \mathrm{d}A^i \right\} \hat{\mu}_t \end{aligned} \tag{61}$$

4 Benchmark model - calibration of financial parameters

The core of the calibration process boils down to pinning down the values for the following 6 parameters/variables: c, b, B, σ_G, i and R. To do that, we impose 6 conditions. The first one is the FOC of the holding company, normalized by the holding equity:

$$\begin{aligned} (p_H QR - 1)\left[1 - F\left(\underline{a}; -\frac{\sigma_G^2}{2}\right)\right] - \frac{p_H c}{\Delta}\left(1 - \phi^m\right)\left[F\left(\bar{a}; -\frac{\sigma_G^2}{2}\right) - F\left(\underline{a}; -\frac{\sigma_G^2}{2}\right)\right] \\ - \lambda^1\left[1 - p_H\left(QR - \frac{B}{\Delta}\right)\right] - \lambda^2\left[1 - \frac{\phi^m p_H c}{\Delta} - p_H\left(QR - \frac{b + c}{\Delta}\right)\right] = 0 \end{aligned} \tag{62}$$

where

$$\bar{a} = i\left[1 - p_H\left(QR - \frac{B}{\Delta}\right)\right] \tag{63}$$

$$\underline{a} = i\left[1 - \frac{p_H c}{\Delta\beta} - p_H\left(QR - \frac{b + c}{\Delta}\right)\right] \tag{64}$$

and

$$\lambda^1 = f\left(\bar{a}; -\frac{\sigma_G^2}{2}\right) i \frac{p_H c}{\Delta}\left(1 - \phi^m\right) \tag{65}$$

$$\lambda^2 = f\left(\underline{a}; -\frac{\sigma_G^2}{2}\right) i \left[p_H QR - 1 - \frac{p_H c}{\Delta}\left(1 - \phi^m\right)\right] \tag{66}$$

On top of it we impose 5 additional constraints. In particular, we want the calibrated model to match selected empirically observed ratios that characterize the banking sector and are relevant for our discussion. The empirical ratios we target are:

1) TARGET 1: Bank operating costs-to-bank assets ratio $BOCA$.

Bank operating costs are given by

$$cI\left[G\left(\bar{A}\right)-G\left(\underline{A}\right)\right]$$

What are banks' assets? We assume that all borrowing made by monitored branches is channeled through banks. Hence we follow the "intermediation" interpretation of HT, rather than the "certification" interpretation. Foreigners deposit money in foreign banks when lending to monitored domestic SOE branches. We also assume that monitoring capital is not enough, i.e. every branch that borrows via banks has to get part of the loan also from depositors. This is implemented by imposing the constraint $\bar{A}+I^m<I$. So total bank lending, or in other words banks' assets, is

$$\int_{\underline{A}}^{\bar{A}}\left(I-A^i\right)\,\mathrm{d}G\left(A^i\right)$$

Therefore, the ratio can be expressed as:

$$BOCA=\frac{cI\left[G\left(\bar{A}\right)-G\left(\underline{A}\right)\right]}{\int_{\underline{A}}^{\bar{A}}\left(I-A^i\right)\,\mathrm{d}G\left(A^i\right)}=\frac{ci\left[F\left(\bar{a}\right)-F\left(\underline{a}\right)\right]}{\int_{\underline{a}}^{\bar{a}}\left(i-a^i\right)\,\mathrm{d}F\left(a^i\right)} \tag{67}$$

2) TARGET 2: Gross foreign bond stock-to-GDP ratio (reported in Figure 2), CBY.

Gross borrowing of branches on the international financial market is associated with the term:

$$CB=\int_{\bar{A}}^{I}I-A^i\,\mathrm{d}G\left(A^i\right)$$

The matched ratio is then, after some substitutions:

$$CBY=\frac{\int_{\bar{A}}^{I}I-A^i\,\mathrm{d}G\left(A^i\right)}{Y}=\frac{K}{Y}\frac{\delta}{p_H Ri\left[1-F\left(\underline{a}\right)\right]}\int_{\bar{a}}^{i}i-a^i\,\mathrm{d}F\left(a^i\right) \tag{68}$$

3) TARGET 3: Gross foreign bank loan stock-to-GDP ratio (reported in Figure 2), BLY.

Gross lending from foreign banks, i.e. bank assets is:

$$BL=\int_{\underline{A}}^{\bar{A}}\left(I-A^i\right)\,\mathrm{d}G\left(A^i\right)$$

The matched ratio becomes, after some substitutions:

$$BLY = \frac{\int_{\underline{A}}^{\bar{A}} \left(I - A^i\right) \, \mathrm{d}G\left(A^i\right)}{Y} = \frac{K}{Y} \frac{\delta}{p_H R i \left[1 - F\left(\underline{a}\right)\right]} \int_{\underline{a}}^{\bar{a}} i - a^i \, \mathrm{d}F\left(a^i\right) \tag{69}$$

4) TARGET 4: Holding leverage (assets-to-equity ratio) *HLev*.

One can count the holding's assets by summing up it's equity with all liabilities. This is simply:

$$K^f + I^m \left[G\left(\bar{A}\right) - G\left(\underline{A}\right)\right] + \int_{\underline{A}}^{\bar{A}} I - I^m - A^i \, \mathrm{d}G\left(A^i\right) + \int_{\bar{A}}^{I} I - A^i \, \mathrm{d}G\left(A^i\right)$$

To obtain leverage, one simply has to divide this expression by holding's equity K^f to get:

$$HLev = 1 + \int_{\underline{a}}^{i} i - a^i \, \mathrm{d}F\left(a^i\right) \tag{70}$$

5) TARGET 5: Bank leverage (assets-to-equity ratio) *BLev*.

Banks' equity is simply equal to

$$K^m = I^m \left[G\left(\bar{A}\right) - G\left(\underline{A}\right)\right]$$

Banks' assets are simply all bank loans:

$$\int_{\underline{A}}^{I} \left(I - A^i\right) \, \mathrm{d}G\left(A^i\right)$$

Bank leverage is then, after minor substitutions:

$$BLev = \frac{\int_{\underline{a}}^{\bar{a}} i - a^i \, \mathrm{d}F\left(a^i\right)}{\frac{p_H c i}{\Delta\beta} \left[F\left(\bar{a}\right) - F\left(\underline{a}\right)\right]} \tag{71}$$

To match these 6 targets, we use a numerical procedure in which we choose the 6 parameters to minimize a certain loss function. The first step of the loss function involves numerically solving the equation 62 for i, given some guessed values for c, b, B, σ_G and R. The second step of the loss function involves computing a GMM-like object which minimizes the following quadratic form:

$$h\left(\boldsymbol{\theta}\right)' \mathbf{W} h\left(\boldsymbol{\theta}\right) \tag{72}$$

where

$$\boldsymbol{h}\left(\boldsymbol{\theta}\right)=\left\{\begin{array}{c}\frac{BOCA-BOCA^{data}}{BOCA^{data}}\\ \frac{CBY-CBY^{data}}{CBY^{data}}\\ \frac{BLY-BLY^{data}}{BLY^{data}}\\ \frac{HLev-HLev^{data}}{HLev^{data}}\\ \frac{BLev-BLev^{data}}{BLev^{data}}\end{array}\right\} \tag{73}$$

where $\mathbf{W}$ is a 5×5 identity matrix.

Note that even though the overall problem involves solving for 6 values with 6 constraints, we cannot obtain a perfect match of the empirical targets. This is for two reasons. First, the system is non-linear. Secondly, the support of the parameter space is bounded. These bounds come from the fact that all parameter and variables have to be strictly positive. On top of that, the parameters have to satisfy a set of "regularity" restrictions: $0<\underline{A}<\bar{A}<I-I^m<I$. We also require that shadow prices $\lambda^1>0$ and $\lambda^2>0$ as well as $p_H>p_L$ and $B>b$. Simple algebra translates this into the following necessary inequalities:

1) $0<\underline{A}$ translates into

$$R<\frac{1}{p_H}+\frac{b}{\Delta}+\frac{c}{\Delta}\left(1-\phi^m\right)$$

2) $\underline{A}<\bar{A}$ translates into

$$B-b-c\left(1-\phi^m\right)>0$$

3) $\bar{A}<I-I^m$ translates into

$$\frac{c\phi^m+B}{\Delta}<R$$

4) $I-I^m<I$ is satisfied if $I^m>0$ or $p_H>p_L$, $c>0$ and $\phi^m>0$.

5) $\lambda^1>0$ translates into

$$\frac{p_H c}{\Delta}\left(1-\phi^m\right)>0$$

6) $\lambda^2>0$ translates into

$$R>\frac{1}{p_H}+\frac{c}{\Delta}\left(1-\phi^m\right)$$

The final restriction, $b+c>B$, is postulated by Holmström and Tirole (1997), p. 674, footnote 10. It implies that monitoring is socially valuable, i.e. that $c\Delta<pH\left(B-b\right)$.

5 Model with only categories 1 and 3

The model now consists of the following 20 variables, all known at time t: D_{t+1}, Ψ_{t+1}, C_t, H_t, W_t, λ_t^h, Y_t, K_{t+1}, r_t^K, Q_t, I_t, $\bar{A}_t$, $\bar{a}_t$, i_t, λ_t^1, X_t, Π_t^f, K_{t+1}^f, μ_{t+1} and CB_t.

The 19 equations are:

$$\Psi_t = \bar{\Psi} + \tilde{\Psi}\left(\mathrm{e}^{\bar{D}_t - \bar{D}} - 1\right) \tag{74}$$

$$C_t + Q_t X_t + \Psi_t R_t^* D_t = W_t H_t + r_t^K K_t + \left(1 - \phi^f\right)\Pi_t^f + D_{t+1} \tag{75}$$

$$\lambda_t^h = \beta^h \Psi_{t+1} R_{t+1}^* E_t \lambda_{t+1}^h \tag{76}$$

$$\lambda_t^h = \left(C_t - \kappa \frac{H_t^\tau}{\tau}\right)^{-\sigma} \tag{77}$$

$$\kappa H_t^{\tau-1} = W_t \tag{78}$$

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha} \tag{79}$$

$$(1-\alpha)\frac{Y_t}{H_t} = W_t \tag{80}$$

$$\alpha \frac{Y_t}{K_t} = r_t^K \tag{81}$$

$$K_{t+1} = (1-\delta)K_t + X_t - \frac{\varphi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{82}$$

$$X_t = p_H R I_t \left[1 - G\left(\bar{A}_t; \mu_t\right)\right] \tag{83}$$

$$Q_t\left[1 + \varphi\left(\frac{K_{t+1}}{K_t} - 1\right)\right] = E_t \beta^h \frac{\lambda_{t+1}^h}{\lambda_t^h}\left\{r_{t+1}^K + Q_{t+1}\left[1 - \delta + \varphi\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\frac{K_{t+2}}{K_{t+1}} - \frac{\varphi}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)^2\right]\right\} \tag{84}$$

$$K_{t+1}^f = \phi^f \Pi_t^f \tag{85}$$

$$\Pi_t^f = p_H Q_t R I_t \left[1 - G\left(\bar{A}_t; \mu_t\right)\right] + \int_0^{\bar{A}_t} A_t^i \mathrm{d}G\left(A_t^i; \mu_t\right) - \int_{\bar{A}_t}^{\infty}\left(I_t - A_t^i\right)\mathrm{d}G\left(A_t^i; \mu_t\right) \tag{86}$$

$$(p_H Q_t R - 1)\left[1 - G\left(\bar{A}_t;\mu_t\right)\right] = \lambda_t^1 \left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] \tag{87}$$

$$\lambda_t^1 = g\left(\bar{A}_t;\mu_t\right) I_t \left(p_H Q_t R - 1\right) \tag{88}$$

$$\bar{A}_t = I_t \left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] \tag{89}$$

$$\bar{a}_t = i_t \left[1 - p_H\left(Q_t R - \frac{B}{\Delta}\right)\right] \tag{90}$$

$$i_t = \frac{I_t}{K_t^f} \tag{91}$$

$$\mu_{t+1} = \ln K_{t+1}^f - \frac{\sigma_G^2}{2} \tag{92}$$

$$CB_t = \int_{\bar{A}_t}^{I_t} \left(I_t - A_t^i\right) \mathrm{d}G\left(A_t^i;\mu_t\right) \tag{93}$$

We also have the following list of 14 parameters: ϕ^f, $\bar{\Psi}$, $\tilde{\Psi}$, β^h, κ, τ, σ, α, δ, φ, σ_G, p_H, p_L, B.

6 Models with only categories 1 and 2

The model consists of the following 23 variables, all known at time t: D_{t+1}, Ψ_{t+1}, C_t, H_t, W_t, λ_t^h, Y_t, K_{t+1}, r_t^K, Q_t, I_t, $\underline{A}_t$, $\underline{a}_t$, i_t, λ_t^2, X_t, Π_t^f, β_t, μ_{t+1}, R_t^m, I_t^m, K_{t+1}^m, K_{t+1}^f and BL_t.

The complete list of 23 equations is:

$$\Psi_t = \bar{\Psi} + \tilde{\Psi}\left(\mathrm{e}^{\bar{D}_t - \bar{D}} - 1\right) \tag{94}$$

$$C_t + Q_t X_t + \Psi_t R_t^* D_t = W_t H_t + r_t^K K_t + \left(1 - \phi^f\right) \Pi_t^f + D_{t+1} \tag{95}$$

$$\lambda_t^h = \beta^h \Psi_{t+1} R_{t+1}^* E_t \lambda_{t+1}^h \tag{96}$$

$$\lambda_t^h = \left(C_t - \kappa \frac{H_t^\tau}{\tau}\right)^{-\sigma} \tag{97}$$

$$\kappa H_t^{\tau-1} = W_t \tag{98}$$

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \tag{99}$$

$$(1-\alpha)\frac{Y_t}{H_t} = W_t \tag{100}$$

$$\alpha \frac{Y_t}{K_t} = r_t^K \tag{101}$$

$$K_{t+1}^m = \phi^m \frac{p_H c I_t}{\Delta}\left[G\left(I_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \tag{102}$$

$$K_{t+1} = (1-\delta) K_t + X_t - \frac{\varphi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{103}$$

$$X_t = p_H R I_t \left[1 - G\left(\underline{A}_t;\mu_t\right)\right] \tag{104}$$

$$Q_t\left[1+\varphi\left(\frac{K_{t+1}}{K_t}-1\right)\right] = E_t \beta^h \frac{\lambda_{t+1}^h}{\lambda_t^h}\left\{r_{t+1}^K + Q_{t+1}\left[1-\delta+\varphi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\frac{K_{t+2}}{K_{t+1}} - \frac{\varphi}{2}\left(\frac{K_{t+2}}{K_{t+1}}-1\right)^2\right]\right\} \tag{105}$$

$$K_{t+1}^f = \phi^f \Pi_t^f \tag{106}$$

$$\begin{aligned}\Pi_t^f = {} & p_H Q_t R I_t \left[1 - G\left(\underline{A}_t;\mu_t\right)\right] + \int_0^{\underline{A}_t} A_t^i \mathrm{d}G\left(A_t^i;\mu_t\right) + \int_{I_t}^{\infty}\left(A_t^i - I_t\right)\mathrm{d}G\left(A_t^i;\mu_t\right) \\ & - \frac{p_H c I_t}{\Delta}\left[G\left(I_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] - \int_{\underline{A}_t}^{I_t}\left(I_t - \frac{p_H c I_t}{\Delta\beta_t} - A_t^i\right)\mathrm{d}G\left(A_t^i;\mu_t\right)\end{aligned} \tag{107}$$

$$\begin{aligned}&\left(p_H Q_t R - 1\right)\left[1 - G\left(\underline{A}_t;\mu_t\right)\right] - \frac{p_H c}{\Delta\beta_t}\left(\beta_t - 1\right)\left[G\left(I_t;\mu_t\right) - G\left(\underline{A}_t;\mu_t\right)\right] \\ & - \frac{p_H c}{\Delta\beta_t}\left(\beta_t - 1\right) g\left(I_t\right) I_t - \lambda_t^2\left[1 - \frac{p_H c}{\Delta\beta_t} - p_H\left(Q_t R - \frac{b+c}{\Delta}\right)\right] = 0\end{aligned} \tag{108}$$

$$\lambda_t^2 = g\left(\underline{A}_t; \mu_t\right) I_t \left[\left(p_H Q_t R - 1\right) - \frac{p_H c}{\Delta \beta_t} \left(\beta_t - 1\right) \right] \tag{109}$$

$$\underline{A}_t = I_t \left[1 - \frac{p_H c}{\Delta \beta_t} - p_H \left(Q_t R - \frac{b+c}{\Delta} \right) \right] \tag{110}$$

$$\underline{a}_t = i_t \left[1 - \frac{p_H c}{\Delta \beta_t} - p_H \left(Q_t R - \frac{b+c}{\Delta} \right) \right] \tag{111}$$

$$i_t = \frac{I_t}{K^f} \tag{112}$$

$$K_t^m = I_t^m \left[G\left(I_t; \mu_t\right) - G\left(\underline{A}_t; \mu_t\right) \right] \tag{113}$$

$$R_t^m = \frac{c I_t}{\Delta} \tag{114}$$

$$I_t^m = \frac{p_H R_t^m}{\beta_t} \tag{115}$$

$$\mu_{t+1} = \ln K_{t+1}^f - \frac{\sigma_G^2}{2} \tag{116}$$

$$BL_t = I_t^m \left[G\left(I_t; \mu_t\right) - G\left(\underline{A}_t; \mu_t\right) \right] + \int_{\underline{A}_t}^{I_t} I_t - I_t^m - A_t^i \, \mathrm{d}G\left(A_t^i; \mu_t\right) \tag{117}$$

We also have the following list of 16 parameters: ϕ^f, ϕ^m, $\bar{\Psi}$, $\tilde{\Psi}$, β^h, κ, τ, σ, α, δ, φ, σ_G, p_H, p_L, c, b.

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